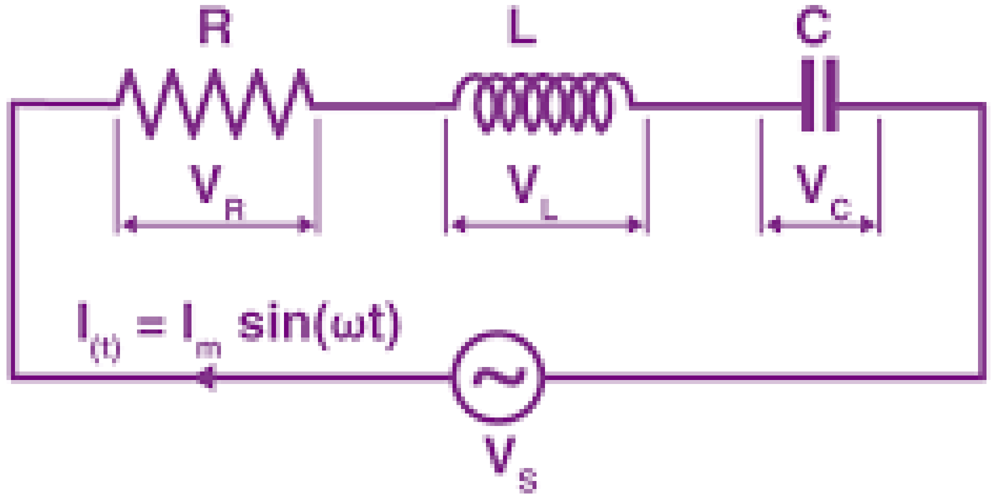
AC Components

YORK COLLEGE & UNIVERSITY CENTRE

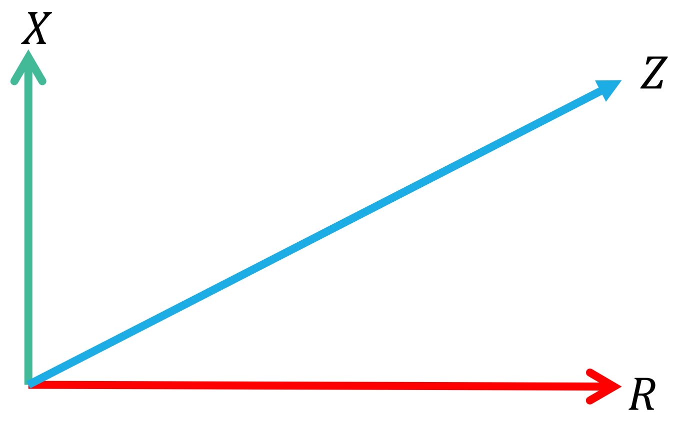
DC vs AC circuit analysis

- In DC circuits we define ohms law as $V = IR$
- However, when working with AC circuits we use $V = IZ$
- Voltage and current are still the same however resistance is replaced with impedance (Z)



Impedance

- Impedance is the Z value in $V = IZ$
- It is the combination of two values:
  - Reactance (X) from capacitors and inductors
  - Resistance (R) from resistors



Impedance

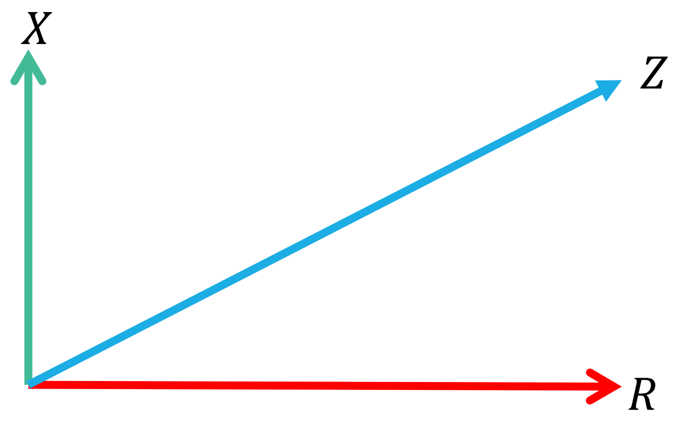
- When calculating impedance, we treat the value as a vector, we combine the X and R value using trigonometry

- This means polar impedance can be calculated using

$$\boldsymbol{Z = \sqrt{R^2 + X^2}}$$

- We can then work out the angle of the polar impedance using $\mathbf{tan^{-1}}(\frac{\boldsymbol{X}}{\boldsymbol{R}})$.

- Though when we do this make sure, we are in radian mode on our calculator

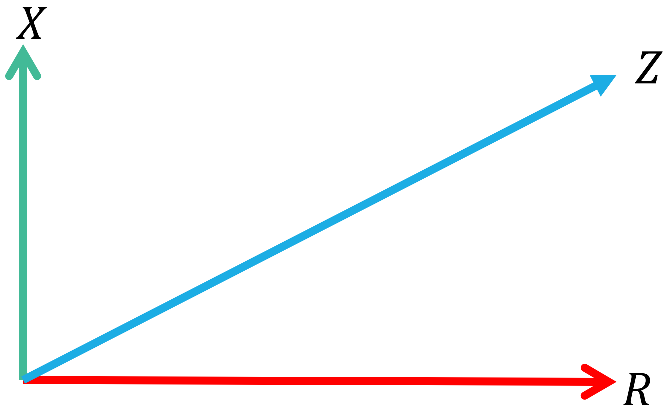


Impedance

- Most of the time when doing complex AC circuit analysis, we leave our impedance in cartesian format making addition easier

- To add two impedances, we just combine the two X components together and then the two R components together

- So $Z_1 + Z_2 = (R_1 + R_2,\ X_1 + X_2)$

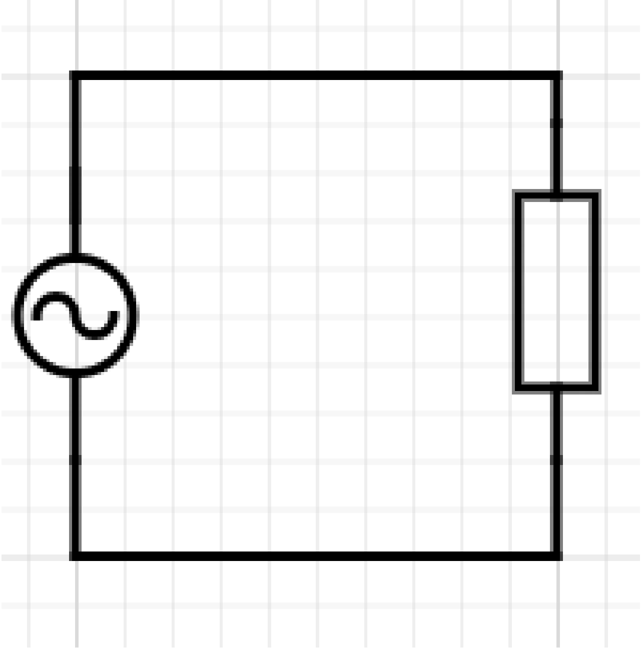


Resistors

- When we have resistors in our circuit, they don't produce a phase shift, this means both voltage and current are aligned

- All they do is limit current reducing our peak current for the circuit

- If we have a purely resistive circuit, we can just use $V = IR$ to calculate our values the same as DC



V = IR @ instantaneous t

$V_{max} = I_{max} * R$

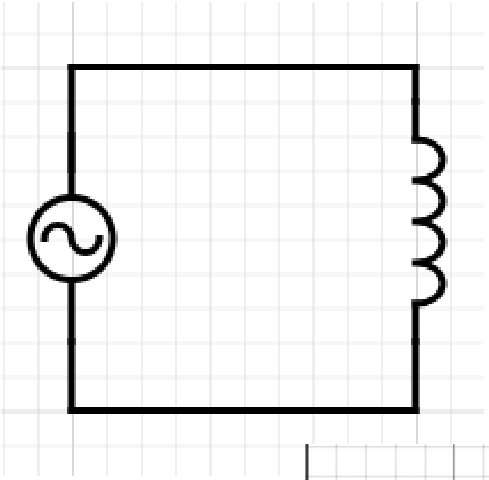
$V_{rms} = I_{rms} * R$

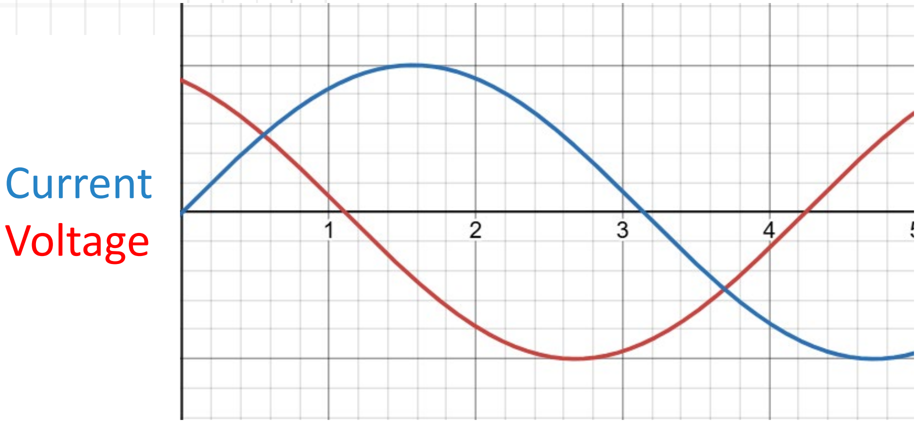
Reactance

- Reactance is the X value in our impedance calculations
- It incorporates all inductors and resistors in the circuit
- Reactance is primarily used to tell us whether current or voltage is leading/lagging
- $+X \rightarrow$ voltage leads (inductive)
- $-X \rightarrow$ current leads (capacitive)

Inductors

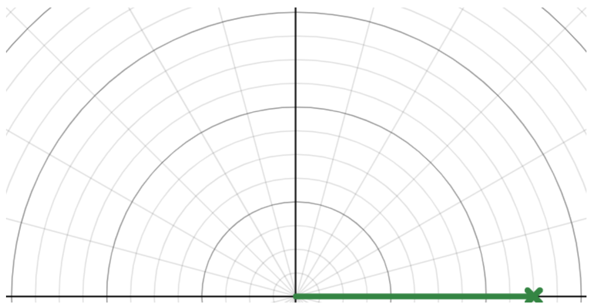
- Inductors cause the change in current to slow down (or phase shift)
- This is because the changing current through the inductor generates a changing magnetic field
- This changing magnetic field induces a voltage back into the inductor opposing the original current





Inductive Reactance

- Inductors generate a **positive** reactance value which acts at $+90°$



- We can work out this reactance value using

$$X_L = \omega L \text{ or } X_L = 2\pi f L$$

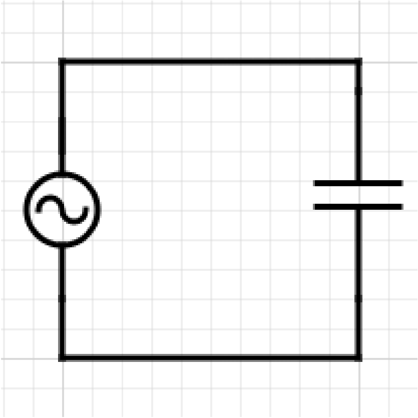
- This means for a purely inductive circuit we have

$$V = I * j\omega L \text{ or } V = I * j2\pi f L$$

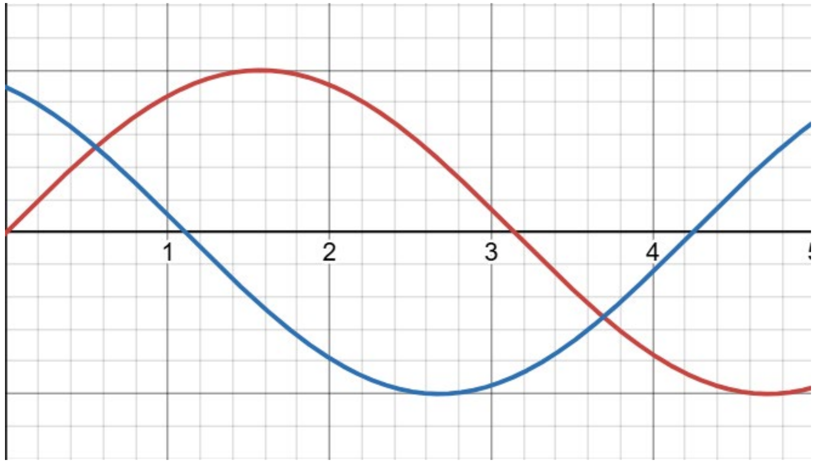
Note we need to include the j here to show we are treating this as an impedance with a direction not just a raw value, the j isn't actually a variable

Capacitors

- Capacitors cause the change in voltage to slow down (or phase shift)
- This is because they take time to charge and discharge
- This causes the voltage to slip behind the current



Current
Voltage



Capacitive Reactance

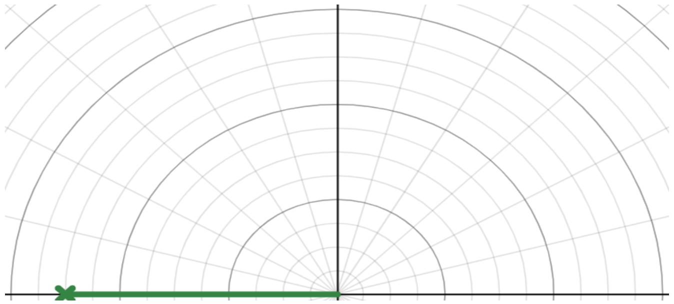
- Inductors generate a **negative** reactance value which acts at -90°

- We can work out this reactance value using

$$X_C = \frac{1}{\omega C} \text{ or } X_C = \frac{1}{2\pi f C}$$

- This means for a purely capacitive circuit we have

$$V = I * -\frac{1}{j\omega C} \text{ or } V = I * -\frac{1}{j2\pi f C}$$



Note we need to include the j here to show we are treating this as an impedance with a direction not just a raw value, the j isn't actually a variable

Your Turn

- Calculate the **capacitive reactance** and then the **impedance in polar form**

$$C = 100\,\mu\text{F}$$
$$C = 47\,\mu\text{F}$$
$$C = 220\,\mu\text{F}$$

- Calculate the **capacitive reactance** and then the **impedance in both cartesian and polar form**

$$R = 10\,\Omega,\ C = 100\,\mu\text{F}$$
$$R = 22\,\Omega,\ C = 47\,\mu\text{F}$$
$$R = 5\,\Omega,\ C = 220\,\mu\text{F}$$

Assume a supply frequency of **50 Hz** for all questions.

Combining Inductors, Capacitors and Resistors (Series)

- For most series AC circuit questions, we can use a universal equation to give us an answer

$$\boldsymbol{Z = R + j(X_L - X_C)}$$

- This of course can then be put into the polar equation or left as it is in cartesian format, to convert to polar we use:

$$|\boldsymbol{Z}| = \sqrt{\boldsymbol{R^2 + (X_L - X_c)^2}} \text{ and } \boldsymbol{tan^{-1}(\frac{(X_L - X_C)}{R})}.$$

Combining Inductors, Capacitors and Resistors (Parallel)

- When we look at parallel components it gets slightly more difficult, we need to look at **admittance** ($\boldsymbol{Y}$) where:

$$\boldsymbol{Y = \frac{1}{Z}}$$

- This means we can work out the **overall impedance** of our parallel lines then find **their admittance**, to then **add them together:**

$$\boldsymbol{Y_{tot} = Y_1 + Y_2 + Y_3 + \cdots}$$

- We can then **convert back** to impedance by doing:

$$\boldsymbol{Z_{tot} = \frac{1}{Y_{tot}}}$$